Why thousands of telecommunications engineers have invested in Mathcad

Ever wondered why some engineers regard their PC almost as another member of the design team? Take a look and you'll probably find their main application is Mathcad.

For too long, too many engineers have been asked to expend too much of their creativity adapting to tools developed for other disciplines. Spreadsheets developed for accountants. "Word processors" developed for administrators.

But there is a tool - indeed, a complete PC environment - developed by engineers with the needs of engineers at its heart. That tool is Mathcad, and it's quietly become one of the most widely-used technical applications in the world.

Mathcad is a technical document creation system, an engineering calculation environment and a mathematical reference library which, with the addition of libraries and on-line resources, can be made even more relevant to individual specialisations such as telecommunications engineering. It's all these things, and more. Mathcad solves mathematical problems, generates graphs and visualisations, and does everything you'd ever dreamed a PC application for engineers would do. If you know how much your time is worth, you'll know it's time to take a look at putting Mathcad at the heart of your work.

This pack uses a few examples to give you a brief introduction to how Mathcad can start to really make your PC work for you. Read Bruce McNair of AT&T describing how Mathcad lets the company simulate a whole system - transmitter, channel, receiver - and helps solve symbolic problems that can't be simulated numerically. Take a look at a sample from one of the Mathcad "Electronic Books" from the telecomms sector. More than a reference, Electronic Books actually perform the operations they document: see how the guide to radio transmission and reception doesn't just describe the process: it actually does the complete calculations.

With continuous input from over 1.5 million users worldwide, Mathcad has been developed into one of the most wide-ranging applications on the PC today. When you've taken a look at this pack, move across to **http://mathcad.adeptscience.com** for more information or call one of our product experts to discuss what it can do for you.

Let's reclaim the PC for engineers.

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AT&T uses Mathcad to simulate and test new technology

As cell phone providers scramble to up their second-generation (2G) bit rates for wireless digital communications from a measly 14 kbits/second to 3G rates of 2 Mbits/s, AT&T's Wireless Systems Research Department is preparing 40-Mbit/s technologies using orthogonal frequency-division multiplexing (OFDM).

"We are still at the simulation stage, but we think that within five years we can introduce fourth-generation wireless connectivity at up to 40 Mbits/s," said AT&T technology consultant Bruce McNair. OFDM zeroes in on the problem that keeps 3G wireless from pushing much past 2 Mbits/s - so-called "intersymbol interference" (ISI). ISI results when the bit rate of a channel is large compared with the delay spread. OFDM works by multiplexing the high-frequency content onto many orthogonal carrier signals that can be broadcast at a lower frequency and then reassembled by the receiver without much interference.

According to AT&T (Murray Hill, N.J.), the lower frequencies used by OFDM have longer transmission times, called symbol times, making them virtually immune to ISI problems. In practice, an OFDM transmitter permits each bit to occupy a frequency-time window, which ensures little or no distortion of the waveform. OFDM first converts its serial input data stream into a set of parallel streams, each of which runs at a much slower bit rate than the combined signal.

For instance, if 8-bit bytes are being transmitted, they could be separated into eight parallel data streams, each of which would run eight times slower since it only takes a single bit from each byte. The parallel data is then modulated onto separate carrier frequencies, each orthogonal to the others - that is, it contains no harmonic content that overlaps the other carrier frequencies (or their harmonics).

Continued over…

Next comes the tricky part. An inverse fast Fourier transform (IFFT) converts the frequencies within the parallel data streams into separate time-domain waveforms, all of which are recombined into a single serial data stream that can be wirelessly transmitted. Because the IFFT generates samples of the waveform whose frequency components are orthogonal, they can be mixed with noise and still be easily separated by the receiver.

"You can't transmit any data without getting noise mixed in at the receiver end," said McNair. "The most troublesome is often multipath noise, resulting from the receiver getting many copies of the same signal that are slightly phase-shifted from one another because they have reflected off buildings or because there is more than one antenna in use."

To simulate the effects of multiple antennas and multipath reflections, McNair uses MathSoft's Mathcad to introduce slightly attenuated and delayed copies of the signals into the mix that is presented to the receiver. Mathcad also introduces some random noise to simulate the other common contributor to transmission errors.

"Mathcad lets us simulate the whole system - transmitter, channel, receiver - and helps us solve symbolic problems that can't be simulated numerically," said McNair.

In the simulation, the receiver performs the opposite steps of the transmitter to recover the original signal. Then it does an FFT on each parallel data stream to recover the frequency-domain information originally encoded by virtue of the pretransmission IFFT. The magnitudes of the frequency components of each parallel data stream correspond to separated components of the original data stream. Demodulation discards the carrier signals for each stream, so the resulting parallel data bits can be recombined into the original stream.

"OFDM virtually solves the multipath problems with high-speed wireless communications, and it can also be precompensated for channel disturbances," said McNair. AT&T believes OFDM systems will effectively eliminate ISI in communications channels right at the source.

Chart Analysis enhanced with Mathcad®

Flexible general-purpose mathematics software can be customized for RF design work

By Alan Victor, IBM

The graphic analysis used in RF circuit design has historically benefitted from the invention of the Smith® Chart [1, 2]. Now, with the advent of math packages such as MATLAB®, Mathematica®, Mathcad® and others, further enhancements are possible. This article reviews classic design examples.

With the capabilities of packages such as Mathcad, a designer can improve productivity and enhance analysis capability with minimum effort. This article discusses the generation of the chart in Mathcad; drawing on the chart within Mathcad; and solving circuit design problems. In addition, Mathcad shows the capability of comparative analysis (for example, device scattering parameter comparisons), modeling parasitics in devices, extracting Q, device stability, mapping, use of the extended chart and other RF design capabilities. Many of these capabilities benefit directly from the ease in which the math package handles complex numbers, coupled with the ease of presenting output in graphic form.

Numerous discussions have occurred within the Mathcad collaboratory and other sites [3, 4] addressing chart construction and techniques. Mathematica is an excellent example of some of the possible RF design techniques [6]. Some of the earlier approaches were too complicated and impeded the real utility of the chart. Either the programming detail was long and inflexible, or the axis titles required in the graphics display mode were so numerous [5] that additional desired data to be displayed was lost in the maze. A return to the fundamental definition and construction of the chart discussed in this article streamlined the process.

Chart construction in Mathcad

Constructing the chart in Mathcad is a three-step process. First, we generate the “real contour circles.” Second, we generate the “single real axis.” Finally, we generate the “reactance contours.”

If the inductance and capacitance regions are treated as separate constructs, four labels are required for graphics output. Although the inductance and capacitance contours are mirror images, the separate constructions have been left intact for clarity. Minimizing the number of required labels keeps the display overhead down and provides space for the desired data to be plotted.

The chart is constructed in a rectilinear system instead of polar, therefore entry into the chart requires conversion from a polar coordinate system to a rectilinear system. Data entry could include circle locations for power gain, noise figure, stability, reflection coefficient circles that maximize output power and generalized feedback mapping contours.

The chart requires manipulation of the complex reflection coefficient G. The location of G will be given as a vector with magnitude and angle. In addition, for general applications, a complete description for the location of G should be provided in the form of a circular location, including a value for center and radius. All of these possibilities are readily handled in Mathcad with the following equations:

$$x = c + r\cos(\theta) \qquad (1)$$

and

$$y = c + ir\sin(\theta) \qquad (2)$$

where i is the imaginary complex number, θ is a running variable from 0 to 2π radians, r is the radius or magnitude of the G circle or reflection coefficient, and c is the center location if a circle contour is required (this will be required for applications, such as gain plots, stability plane locations and noise contours). When the chart is plotted in Mathcad and no bounds are set, all four quadrants of circular contours are created. This extended chart is useful for studying designs with G greater than unity. The extended chart expands past the Smith chart unit circle and includes negative real resistance.

Continued over…

The case normally encountered in using the Smith chart for passive networks and non generative circuit cases contains only the unit circle and all the reactance contours terminate on this circle. The addition of control statements in Mathcad could constrain reactance contours so that they terminate on the unit circle. For this article, we have clipped the chart as required to contain whatever region may be of interest.

Extended chart

In Mathcad, the chart is simply bounded and "clipped" to unity as illustrated in Appendix B. If negative resistance regions or stability analysis requires extending the chart, then the "clip" is extended or removed. In Figure 3, the boundary is extended to twice the unit circle radius.

Additional negative resistance contours are added by extending the R array table, as shown in Figure 4 and extending the "clip area." The X array table is also increased for more detail and a popular sequence set is in a 1-2-5 format. Or, the table can be set to match any of the displays associated with a VNA such as the HP8753 series. In Figure 4, Γ is increased to 10 and additional negative resistance contours are exposed. If the same chart clips to Γ = 1, then the unit circle will be detailed.

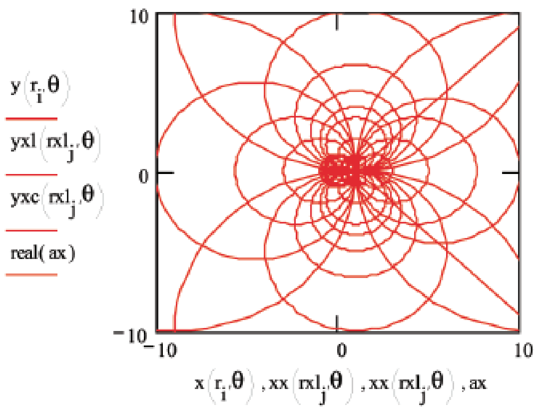

Figure 4. Extended Smith chart: Γ = 10.

Going around in circles

Circle plotting requires only the entry of center location and radius. For example, the location of constant VSWR contours requires a value of Γ that varies with radius proportional to an impedance, and a center at the origin of the chart. The equations and entry into Mathcad are shown. This same concept is extended for computing and plotting noise contours, stability planes, feedback and n-port mapping, and evaluating component performance, for example *Q*. This is illustrated with the example in Appendix C that evaluates a device from the measured data section.

The SWR is assigned to an array table consisting of four fixed values. The Γ associated with each SWR value is calculated and the circle swept and *x, y* coordinated values entered into the chart. This construct could have been any type of contour. In the specific case of noise figure [7], the location of the circles would be geocentric with respect to F_{min} and the r_n of a device and at a specific angle set by the noise parameters associated with that device.

Evaluation of device from measured data

A set of *Q* contours are added to the chart by connecting all equal R-X contours. This would include the capacitive and inductive circles as they intersect the corresponding R values. Two circles are located with a radius of 1.414 and centers at normalized R = 1 and normalized jX = +1 and –1 [8]. With circular arcs drawn on the chart, this construct is used to evaluate component loaded *Q*.

The scattering data table is read from the VNA imported to the chart program and then plotted with the chart as an overlay. The component *Q* of a shunt configured-tuned inductor with parallel shunt parasitic capacitance is determined from the 3 dB bandwidth equation.

The "square" data points intersect the *Q* contours at the –3 dB power point, and f_0 is located as Γ crosses the real axis (diamond tick). The clockwise contour sweep is indicative of a parallel resonance network. A mirror image of this contour exists for the dual, series resonant structure.

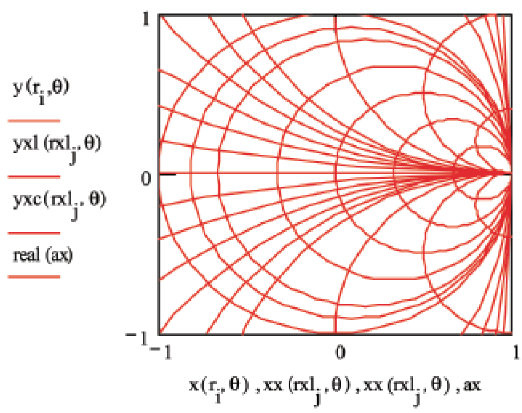

Figure 5. Unit Smith chart: Γ = 1.

Scattering data is imported as a tabular array, plotted a Γ coefficient, and markers added where the one port reflection coefficient intersects or cuts the Q-contour shown in Appendix D on page 60). The markers are

Continued over…

associated with a table of Γ as well as frequency. The Q is computed directly from the definition of $\Delta f/f_0$ within Mathcad and is illustrated in Figure 8. A data array read from the VNA and consists of 200 data points. Either one port S-parameters for a shunt measured element or two port measurement of a series configured element may be used. The two port data is manipulated Mathcad and converted to an equivalent set of one port reflection coefficient set and Γ plotted.

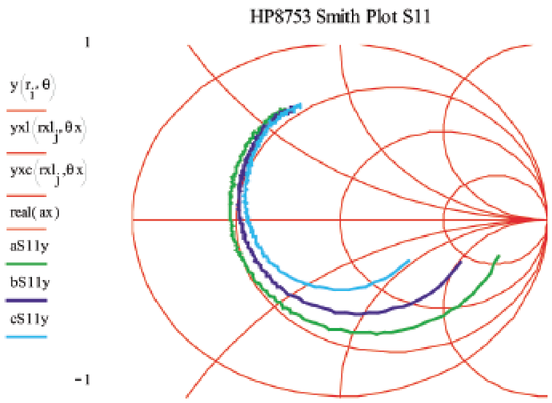

Figure 9. The result of increased collector current at constant collector voltage for a bipolar device over a 6 GHz frequency span.

The center frequency data point is noted as the diamond tick in Figure 8 (shown in Appendix E) and the –3 dB power points are noted as square tick markers.

Comparative analysis

Scattering data is imported to Mathcad either via the serial port or a text file from a VNA. Other file formats such as CITI are also acceptable because they can be easily modified to a text file and placed into a "Touchstone format." The previous example illustrated the evaluation of the Q of a passive component. Measured data from the VNA is imported to Mathcad, and using Smith chart construction techniques in Mathcad, unloaded Q is evaluated. Another powerful technique is the comparative analysis of scattering data plots for devices operating at different voltages or currents.

Comparative plots for devices during design and development are useful for gaining insight into device models and sensitivity. The read file command in Mathcad provides easy, simultaneous inspection of device parameter variations. Up to 12 different plots may be added to the chart and grouped for comparative analysis. For example, the following arrangement is used to compare input impedance and any parasitic resonance in a device as a function of current. Scattering data is directly read from the VNA and imported to construct routine and is displayed. The data array is 201 elements long and in standard Touchstone format, magnitude and angle.

$$end := rows(a)$$

$$rows(a) = 201$$

$$n := 1 .. end$$

$$aS11x_n := \left(a^{\langle 2\rangle}\right)_n \times \cos\left[\left[\left(a^{\langle 3\rangle}\right)_n \times \deg\right]\right]$$

$$aS11y_n := \left[\left(a^{\langle 2\rangle}\right)_n\right] \times \sin\left[\left[\left(a^{\langle 3\rangle}\right)_n \times \deg\right]\right]$$

Entry to the chart requires conversion to rectilinear coordinates. The plots below show the result of increased collector current at constant collector voltage for a bipolar device over a 6 GHz frequency span.

The increasing frequency sweep is clockwise. The device package shows a resonance below 6 GHz, and the low frequency performance (for example, the input impedance) is typical. An increase in r_e occurs as the total emitter current is decreased. With increasing frequency, the input impedance is dominated by the package parasitic, and at the end of the frequency sweep, no discernible difference is seen.

Conclusion

The Smith chart construct in circuit design and analysis is a powerful tool and graphics aid. Complementing this with a mathematics package such as Mathcad makes a desirable analytic and design tool. The designer can quickly verify and validate models and designs and can add additional mathematical processes to the system. This capability is not readily possible in many of the current RF simulation and design packages available.

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Continued over…

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8. Microwave Network Analyzer Applications, Hewlett Packard AN117-1, June 1970.

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APPENDIX A

R table

$i := 1..5 \quad \theta := 1, .1..2 \times \pi$

Circle sweep for conductance and reactance contours, .01 radian step should give fine resolution.

$r_i :=$

0
.333
1
3

The table vector array describes *r*. Four values are used to generate the conductance circles at 150 ohms, 50 ohms, 16.67 ohms and the unit circle. The *r* table is input as a vector table and entries are made by inputting the first value followed by a comma for additional values.

$$(r,\theta) := \frac{r}{1+r} + \frac{1}{1+r} \times \cos(\theta)$$

$$(r,\theta) := \frac{1}{1+r} \times \sin(\theta)$$

x and *y* values generate the circles of conductance.

$ax := -1..1$

$\text{real}\,(ax) := 0$

This sequence forms the real axis.

X table

$j := 1..3$

Same θ sweep used here, but a different angle sweep is permitted to achieve more or less resolution than the real axis.

$rxl_j :=$

$$xx(rxl,\theta) := 1 + \frac{1}{rxl} \times \cos(\theta)$$

$$yxl(rxl,\theta) := 1 + \frac{1}{rxl} + \frac{1}{rxl} \times \sin(\theta)$$

$$yxc(rxl,\theta) := 1 + \frac{-1}{rxl} + \frac{1}{rxl} \times \sin(\theta)$$

.5
1
2

This set of sequences generates the X_c and X_L reactance circles. Note that *xx*() is used for both contours and that the two other contours differ by a sign change. Again, *rxlj* is a vector table loaded with values for three reactance contours: 100 ohms, 50 ohms, 25 ohms.

These tables can be extended and changed to add more detail to the chart. The R and X tables used here construct a chart that emulates the default chart used on the HP8753 series VNA. The chart provides for the inscription of a 3:1 VSWR circle.

Continued over…

The charts are limited in x and y dimension by clipping all values to +1 and -1.

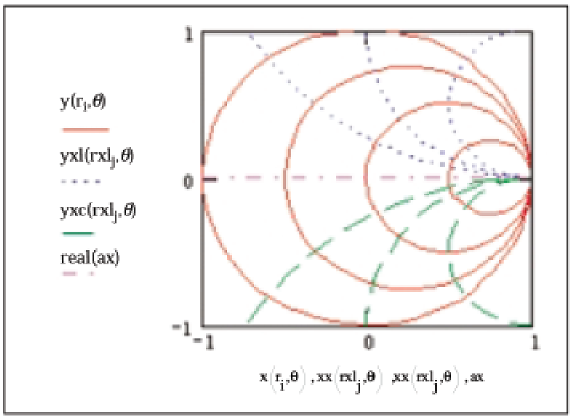


Figure 1. All four traces are shown.

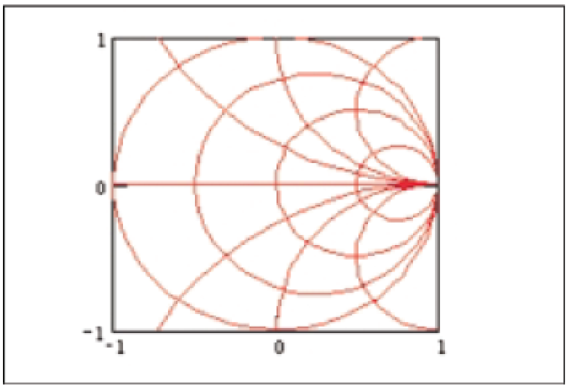


Figure 2. All traces are set to red; legends and labels are turned off.

APPENDIX B

Circles on the chart

Though the chart appears polar, the entry is in rectilinear. The chart is sitting on a coordinate system bounded by $x = (1-, 0, +1)$ and $y = (1-, 0, +1)$. To place a circle anywhere on the unit circle or outside the unit circle to the extended chart, use the following:

$\alpha := 0, .1..2 \times \pi$

$\beta := 1.2$ radius

$\delta := 45$ degrees center location

The values above could represent a stability circle location. Using an extended chart, the entire (un)stable locus is located and provides a technique for mapping feedback networks useful in oscillator design.

Convert the center location into a (x, y) pair.

$xc := \beta \times \cos(\delta)$

$yc := \beta \times \sin(\delta)$

Create the circle on the chart.

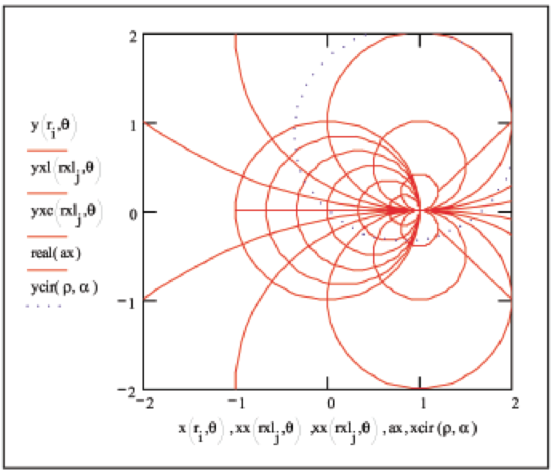


Figure 3. Charting circles.

$xcir(\beta, \alpha) := \beta \times \cos(\alpha) + xc$

$ycir(\beta, \alpha) := \beta \times \sin(\alpha) + yc$

Continued over…

APPENDIX C

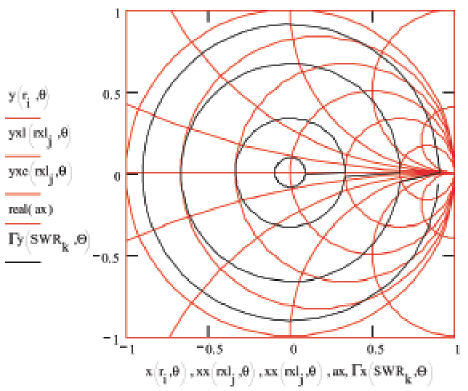
Select Circles or VSWR Plots:

$k := 1 .. 4 \qquad := 0, .1 .. 2$

$$\Gamma x(SWR, \Theta) := \frac{SWR - 1}{SWR + 1} \cdot \cos(\Theta) \qquad y(SWR, \Theta) := \frac{SWR - 1}{SWR + 1} \cdot \sin(\Theta)$$

$$SWR_k := \begin{bmatrix} 1.2 \\ 2 \\ 5 \\ 20 \end{bmatrix}$$

VSWR Contours



APPENDIX D

Generating *Q* circles for the chart

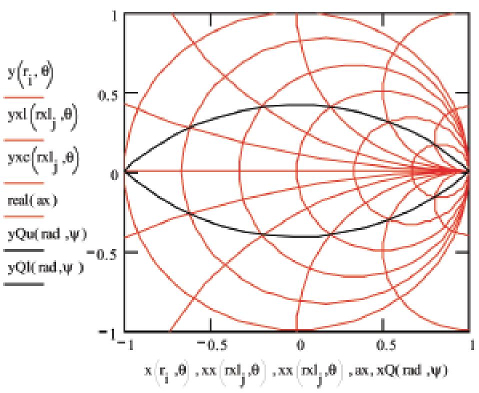
The Q contours intersect all points where $R = \pm jX_C$ or $\pm jX_L$. The center of such a circle is at $x = 0$, $y = 1$, and the radius is 1.414 or root (2).

$$\psi := 0, 0.1 .. 2 \qquad rad := \sqrt{2} \qquad cent := 1$$

$$xQ(rad, \psi) := rad \cdot \cos(\psi) \qquad yQu(rad, \psi) := rad \cdot \sin(\psi) - 1$$

$$yQl(rad, \psi) := rad \cdot \sin(\psi) + 1$$

Smith Chart with Q contours



APPENDIX E

As a test of the Q chart, input a set of measured *S*-parameters with a given reflection coefficient Γ. f_o is 1.9 GHz and *Q* unloaded is to be determined.

$Q := READPRN(\text{"a:QRESP.txt"}) \qquad nend := rows(Q) \qquad nend = 200 \qquad GHz := 1$

$q := 1 .. nend$

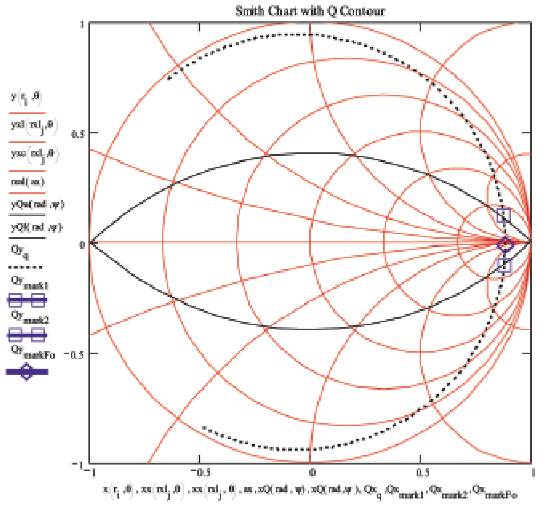
$$Qx_q := \left(Q^{\langle 2 \rangle}\right)_q \cdot \cos\left[\left(Q^{\langle 3 \rangle}\right)_q \cdot deg\right] \qquad Qy_q := \left(Q^{\langle 2 \rangle}\right)_q \cdot \sin\left[\left(Q^{\langle 3 \rangle}\right)_q \cdot deg\right]$$

$mark1 := 48 \qquad freq1 := \left(Q^{\langle 1 \rangle}\right)_{mark1} \qquad freq1 = 1.799 \cdot GHz$

$mark2 := 55 \qquad freq2 := \left(Q^{\langle 1 \rangle}\right)_{mark2} \qquad freq2 = 1.992 \cdot GHz$

$markFo := 52 \qquad freq3 := \left(Q^{\langle 1 \rangle}\right)_{markFo} \qquad freq3 = 1.91 \cdot GHz$

$$Q := \frac{freq3}{freq2 - freq1} \qquad Q = 9.87$$

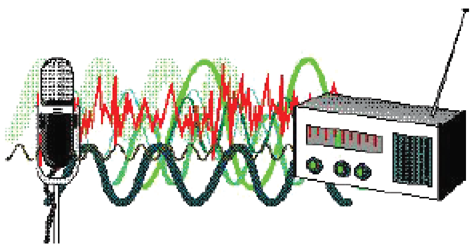


mathcad

Visit our Mathcad website at **http://mathcad.adeptscience.com** for more information or call one of our product experts to discuss what it can do for you.

Real World Maths – AM/FM Radio

Radio is pervasive in our lives; we get much of our news, entertainment, and cultural awareness from listening to the "airwaves." Did you ever wonder how all this information gets sent? This section uses Mathcad to demonstrate the mechanics of radio transmission and reception.



To understand radio, it's important to understand the wave nature of sound. Sound is a series of pressure waves that make air vibrate at different rates (frequencies). These frequencies are interpreted by the ear as different tones. Slow, or low frequencies correspond to low-pitched sounds, and fast or high frequencies to high-pitched sounds.

If sound is to be stored or transmitted electronically, these pressure wave frequencies must be represented as electronic waves. The human ear can hear between about 10 Hz and 20 kilohertz (kHz). Although it's uncommon to find an electronic system which can produce this full range, or bandwidth, with fidelity, let's use this range in this discussion.

Note that a hertz (unit of frequency) is equal to

$$\mathrm{Hz} = 1\frac{1}{\mathrm{sec}} \qquad \mathrm{kHz} \equiv 10^{3}\cdot\mathrm{Hz}$$

Human hearing bandwidth: $\mathrm{bw} := 20\cdot\mathrm{kHz}$

Sound Waves in Time

Examine what an (imaginary) sound wave might look like. It will be a sum of sinusoidal waves of different frequencies between 10 Hz and 20 kHz. Let's pick a relatively high-pitched set.

$$f_1 := 2500\cdot\mathrm{Hz} \qquad \omega_1 := 2\cdot\pi\cdot f_1 \qquad \mathrm{ms} := 10^{-3}\cdot\mathrm{sec}$$

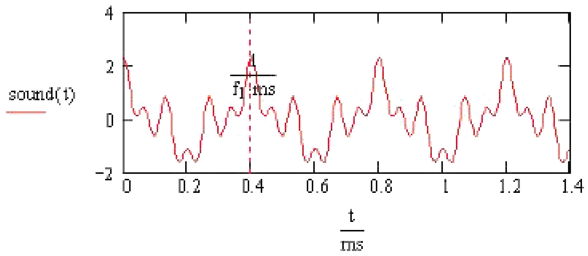
$$\mathrm{sound}(t) := \cos(\omega_1\cdot t) + .7\cdot\cos(3\cdot\omega_1\cdot t) + .6\cdot\cos(6\cdot\omega_1\cdot t)$$

$$t := 0\cdot\mathrm{ms}, \frac{.05}{\omega_1} .. \frac{7\cdot\pi}{\omega_1}$$

Notice that:

$$\frac{3\cdot\omega_1}{2\cdot\pi} = 7.5\,\mathrm{kHz} \qquad \frac{6\cdot\omega_1}{2\cdot\pi} = 15\,\mathrm{kHz}$$

Notice that all of the frequencies chosen fall between 10 Hz and 20 kHz. This sound wave is represented in time (t). If we plot it over time, we'll see the oscillation as the wave travels.



Sound Waves in Frequency

Another useful way to represent a sound wave, or any other kind of wave, is in the **frequency domain**: the wave is represented as a spectrum of frequency components. These frequency components have been written specifically above, so we know them. But, if we didn't, it would be possible to recover them using the Fourier transform. The Fourier transform coefficients are given by a built-in Mathcad function, the fast Fourier transform, or **FFT**.

Number of samples to take:

$$\mathrm{samples} \equiv 2^{11} \qquad k := 0 .. \mathrm{samples} - 1 \qquad \mu s := 10^{-6}\cdot\mathrm{sec}$$

Time between samples:

$$\mathrm{step} := \frac{1}{f_1\cdot\mathrm{samples}} \qquad \mathrm{step} = 0.195\,\mu s \qquad \frac{1}{\mathrm{step}} = 5.12\times10^{3}\,\mathrm{kHz}$$

Time vector:

$$\mathrm{time}_k := \mathrm{step}\cdot k$$

Sound vector:

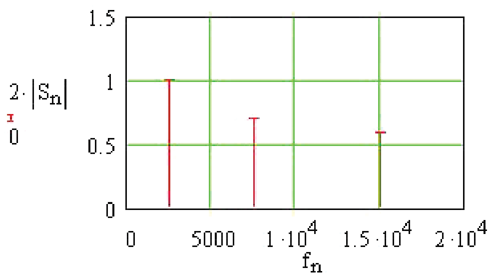
$$\mathrm{signal}_k := \mathrm{sound}(\mathrm{time}_k)$$

Fast Fourier transform:

$$S := \mathrm{FFT}(\mathrm{signal}) \qquad \mathrm{nmax} := \mathrm{length}(S) - 1 \qquad n := 0 .. \mathrm{nmax}$$

Vector of frequencies:

$$f_n := \frac{n}{\mathrm{samples}}\cdot\frac{1}{\mathrm{step}}$$



The trace type "error" has been used to create the spectrum lines.

Continued over…

There is a single spike for each frequency in the sound wave above. If you look at the y-axis, you'll see that the height of each spike is the specified height of that corresponding frequency in the expression for sound(t). If you change the frequencies used in sound(t), or the relative magnitude of each of the terms, you'll see these changes reflected in the spectrum, as well as in the time plot.

Broadcasting the Sound

Radio broadcasting is essentially an exercise in attaching sound waves, such as the one above, to a suitable carrier (broadcasting) frequency, transmitting the carrier with the attached sound, and then removing the carrier at the receiver. Since the sound is already stored electronically using some set of frequencies, why bother with the carrier? In order to transmit or receive sound through the air, an antenna is required. The physics of antenna design require that, for a low frequency signal, on the order of 10 Hz, the required antenna is several miles long! This would certainly be difficult to install in your home or car. Additionally, if your radio had an antenna designed for 60 Hz, you'd probably spend most of your time listening to the hum of power transmission lines. These, and other considerations, such as system bandwidth, make a carrier frequency a necessity.

So, a manageable antenna length requires a high frequency carrier, and one that is much higher in frequency than the sound wave components. This last requirement will insure that the sound frequencies form a little bunch around each carrier frequency, so that multiple carrier frequencies can be used. This allows you to have multiple radio stations. Appropriate frequencies would start around 10 times the bandwidth:

$$f_{min} := 10 \cdot bw \qquad f_{min} = 200\,kHz$$

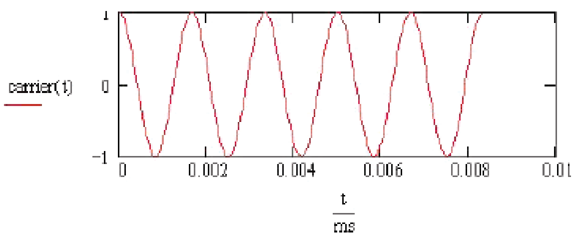
There are two common methods of carrier modulation: combination of the sound with the carrier frequency. First consider amplitude modulation, or AM.

Mixing the Sound and the Carrier

Examine a pure carrier frequency typical for AM radio:

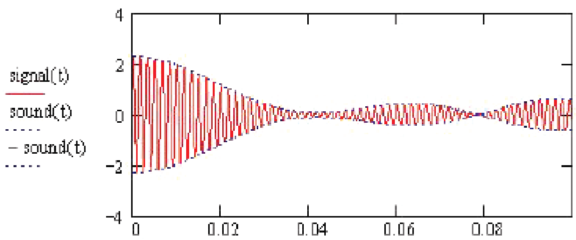
$$f_c := 600 \cdot kHz \qquad \omega_c := 2 \cdot \pi \cdot f_c$$

$$carrier(t) := \cos(\omega_c \cdot t) \qquad t := 0 \cdot sec, \frac{.25}{\omega_c} .. \frac{10 \cdot \pi}{\omega_c}$$



Now consider the following simple multiplication, and the resulting signal:

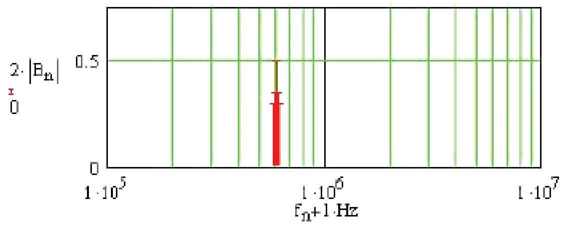
$$signal(t) := sound(t) \cdot carrier(t) \qquad t := 0 \cdot sec, \frac{.2}{\omega_c} .. \frac{\pi}{2 \cdot \omega_1}$$



You can clearly see what happens when there is a large (>10X) difference between the carrier and sound frequencies. The carrier frequency is **amplitude modulated** by the sound. That is, the carrier continues to oscillate in a recognizable sinusoidal pattern, but its amplitude, or height, moves up and down inside an **envelope** defined by the sound wave. The electronic multiplication is performed by a device called a mixer. The sound can then be transmitted on this high frequency wave by an appropriate antenna, received, and unmixed, which will be demonstrated below.

You might be able to anticipate what has happened in frequency. The same components observable in the original sound wave are still there, but they have been shifted up around the carrier frequency.

$$broadcast_k := signal(time_k) \qquad B := FFT(broadcast)$$



This should give you an inkling about how many radio stations can be transmitted at once. If every broadcast frequency is at least 10 times as large as the highest sound frequency, then the broadcast frequencies could be placed apart by just a little more than the sound bandwidth, and each could carry a full complement of sound without interference. This is why station numbering isn't exactly sequential. The station number corresponds to a frequency, and there has to be a little "space" between them.

FM: Phase Locking the Sound and the Carrier

The alternate, and somewhat more robust, method of combining the carrier and the sound is to find the difference in phase between the carrier and the sound wave at any particular moment in time. This is done with an electric circuit known as a phase-locked loop. The effect is to modulate the *frequency* of the carrier wave, rather than the amplitude, and hence is known as **frequency modulation** (FM), as demonstrated below.

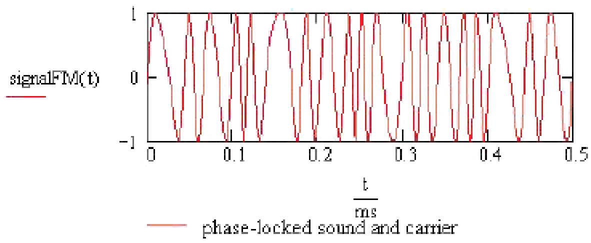
$$f_{cFM} := 40 \cdot kHz \qquad \omega_{cFM} := 2 \cdot \pi \cdot f_{cFM}$$

$$signalFM(t) := \cos(2 \cdot \pi \cdot f_{cFM} \cdot t + mi \cdot sound(t))$$

$$mi \equiv 2$$

$$t := 0 \cdot sec, \frac{.1}{\omega_{cFM}} .. \frac{40 \cdot \pi}{\omega_{cFM}}$$

mi is the modulation index. It has been exaggerated here to display the frequency modulation more clearly.



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An unreasonably small carrier frequency and an excessively large modulation index are used as an example, so that you can see the frequency modulation easily. The waves get very close together in some places, and far apart in others. With realistic values of carrier frequency, between 88 MHz and 108 MHz, the phase shift introduced by the sound is minimal, typically less than ±5 degrees.

You may have noticed that FM stations are much higher in frequency than AM stations. The demonstration above shows you the reason. If sound(t) distorted the signal as much as it does in the graph above, the sound would be unrecoverable, because the carrier frequency would be indistinguishable. Sufficiently high frequencies must be picked, along with the modulation index, to insure that the phase shifts are always small, but still detectable by your radio.

Recovering the Sound

To get the sound back out of an AM wave, it's necessary to multiply it by the carrier frequency again. Analytically,

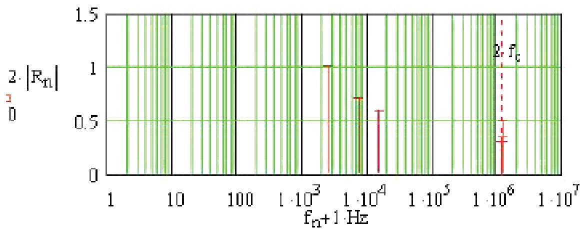
$$\mathrm{sound}(t)\cdot\mathrm{carrier}(t)\cdot\left(2\cdot\cos\left(f_c\cdot t\right)\right) = \mathrm{sound}(t)\cdot\left[\cos\left(f_c\cdot t\right)\cdot\left(2\cdot\cos\left(f_c\cdot t\right)\right)\right]$$

$$2\cdot\mathrm{sound}(t)\cdot\cos\left(f_c\cdot t\right)^2 = \mathrm{sound}(t) + \mathrm{sound}(t)\cdot\cos\left(2\cdot f_c\cdot t\right)$$

This decomposes, as you can see, into the original sound and a cosine term at twice the carrier frequency. If you lowpass filter this signal, i.e., allow the low frequencies to pass through, all that remains is the original sound(t).

Examine the received signal in the frequency domain to see if the analytical expression is confirmed. What you should see is a signal at twice the frequency, with the sound wave frequencies bunched around it, and the original sound frequencies in their proper locations.

$$\mathrm{receive}_k := 2\cdot\cos\left(\omega_c\cdot\mathrm{time}_k\right)\cdot\mathrm{signal}\left(\mathrm{time}_k\right) \qquad R := \mathrm{FFT}(\mathrm{receive})$$



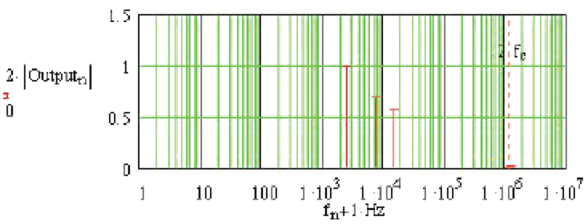
Filtering

A lowpass filter will have the mathematical form:

$$\mathrm{filter}\left(\omega,\omega_c\right) := \frac{\omega_c}{10j\cdot\omega + \omega_c}$$

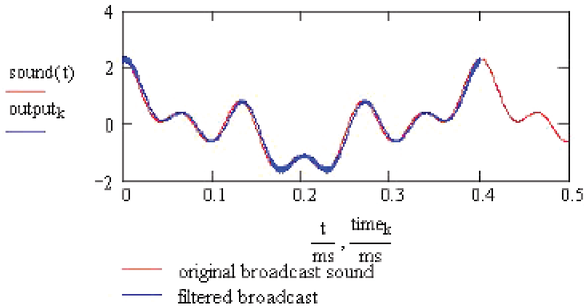
yielding a filtered output:

$$\mathrm{Output}_n := \mathrm{filter}\left(2\cdot\pi\cdot f_n,\omega_c\right)\cdot R_n$$



It is possible to improve the filter significantly with more electronics (more mathematical terms), but this is typical of what will result. There will be some small residual signal at twice the carrier frequency. To see if we've recovered the original signal adequately, find the inverse Fourier transform through the use of another built-in function, **IFFT**:

$$\mathrm{output} := \mathrm{IFFT}(\mathrm{Output})$$



Pretty good! The red line is the received output, and the blue, dotted line is the original sound input. The "fluffiness" on the output signal is an example of noise. This is why radios (particularly AM radios) sometimes have static on the signal. In practical systems, noise is partly the result of attenuation and distortion in the electronics, and partly the limitations on filtering. The output is also ever so slightly phase shifted (occurs later in time). This is also the result of filtering.

As a final test of your understanding, let's look at how a radio tuner would function. Start with a few different sound waves, or "songs," and put each one on a different carrier frequency.

$$\mathrm{freq} := 2347.30\cdot\mathrm{Hz} \qquad \omega := 2\cdot\pi\cdot\mathrm{freq}$$

$$a := 1,3\,..\,7 \qquad b := 2,4\,..\,8$$

$$\mathrm{song1}(t) := \sum_a \frac{1}{a}\cdot\sin(a\cdot\omega\cdot t) \qquad \mathrm{song2}(t) := \sum_b \frac{1}{b}\cdot\sin(b\cdot\omega\cdot t)$$

$$\mathrm{song3}(t) := \cos(\omega\cdot t)^3$$

Carrier frequencies (stations):

$$f_{c1} := 550\cdot\mathrm{kHz} \qquad f_{c2} := 780\cdot\mathrm{kHz} \qquad f_{c3} := 1010\cdot\mathrm{kHz}$$

$$\omega_{c1} := 2\cdot\pi\cdot f_{c1} \qquad \omega_{c2} := 2\cdot\pi\cdot f_{c2} \qquad \omega_{c3} := 2\cdot\pi\cdot f_{c3}$$

To perform the transforms, create a new time step and time vector corresponding to the new base frequency for the songs:

Time between samples:

$$\omega_{c1} := 2\cdot\pi\cdot f_{c1}$$

Time vector:

$$t_k := \mathrm{step}\cdot k$$

Frequency vector:

$$f_n := \frac{n}{\mathrm{samples}+1}\cdot\frac{1}{\mathrm{step}}$$

All the frequencies are broadcast together over the air:

$$\mathrm{airwaves}_k := \cos\left(\omega_{c1}\cdot t_k\right)\cdot\mathrm{song1}\left(t_k\right) + \cos\left(\omega_{c2}\cdot t_k\right)\cdot\mathrm{song2}\left(t_k\right) + \cos\left(\omega_{c3}\cdot t_k\right)\cdot\mathrm{song3}\left(t_k\right)$$

You tune your dial to select a single station. Only this frequency is used to multiply the incoming signal.

Continued over…

$$receive_k := \cos(2\cdot\pi\cdot station\cdot t_k)\cdot airwaves_k \quad R := FFT(receive)$$

yielding a filtered frequency output:

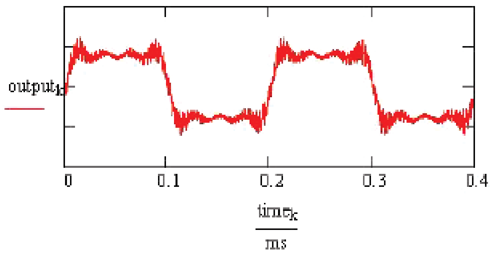
$$Output_n := filter(2\cdot\pi\cdot f_n, 2\cdot\pi\cdot 550\cdot kHz)\cdot R_n$$

The recovered time signal is:

$$output := IFFT(Output)$$

Select a radio channel by changing the value of station below. The signal carried on that station will be displayed below. The available AM stations on your "dial" are **550 kHz**, **780 kHz**, and **1010 kHz**. Try values between these stations too, to see the result. It will be visually equivalent to static. What happens when you get close to a station value, but not on it?

$$station \equiv 550\cdot kHz$$



These look a lot sloppier than the single station example. The reason is interference from the high frequency carriers of the other stations which are not completely filtered. As mentioned earlier, you can improve the filter in lots of ways, electronically, and so most radios are not this noisy.

AM radio carries lots of information these days; for example, you can listen to a baseball game on a summer day at the beach. To understand the player stats that get broadcast, see the section on baseball statistics.

Reference

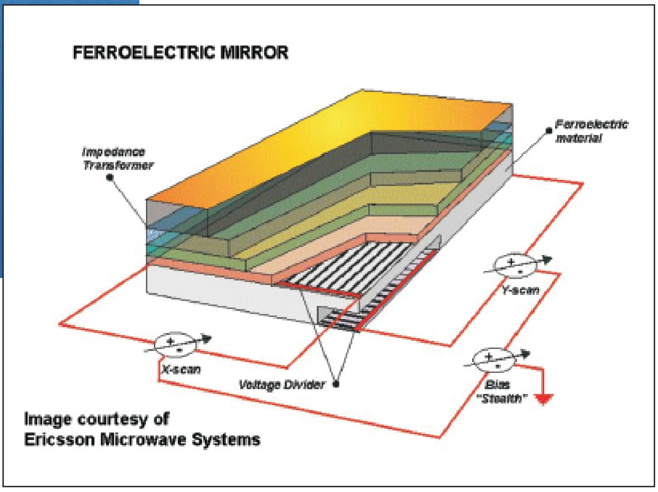
1. William Siebert, Circuits, Signals and Systems, MIT Press, Cambridge, MA, 1986.

Mathcad Simulates Smartskin...

Saab 340 command aircraft equipped with the ERIEYE Airborne Early Warning & Control (AEW&C) system from Ericsson Microwave Systems.

The Ericsson Group is a world-leading telecommunications supplier, whose core company for microwave communications and defence electronics is Ericsson Microwave Systems. With 4700 employees at its headquarters in Mölndal, Sweden, Ericsson Microwave has 130 Mathcad users.

One Mathcad application at Ericsson has been the analysis of its 'smart skin' technology for radar screening of aircraft and ground vehicles. A layered ceramic coating only 1.5 cm thick, this contains a 'ferroelectric mirror' whose electrical properties are controlled by an applied voltage. This varies the radar reflection characteristics of the coating. Reflected phase, direction and amplitude can all be altered, returning false information that confuses a radar scan.



'Smart skin' electronic coating from Ericsson Microwave.

Kent Falk, an engineer at the Ericsson Microwave Systems Innovation Centre, explains: "To calculate the radar reflection, we used Mathcad to analyse the coating as a microwave transmission line, with the dielectric constant varying with depth. This led to the Riccati equation - y' = p(x) + q(x)*y + r(x)*y^2 - for which we used the rkadapt differential equation solver."

This was a 'power application' of Mathcad. First Kent derived the profile of dielectric constant within the layers. This highly complex formula made intensive use of Mathcad's Greek symbol set; its natural layout for algebra; and its programming language to define the electrical discontinuities in the material.

Then he applied Mathcad's adaptive solver rkadapt to solve the electric field strength to 100 steps. The result was a 3D graph showing how radar reflection varied with frequency and applied voltage.

"This took over a day to calculate on a 266MHz Pentium," Kent says. "While the program execution is somewhat

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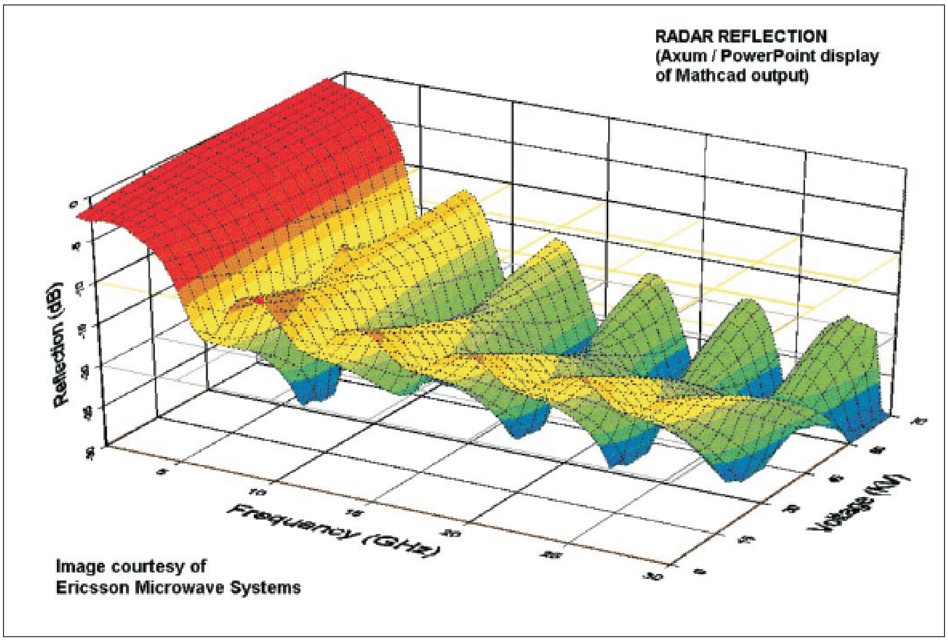

Applied voltage alters the radar reflection of the 'smart skin' (Axum / PowerPoint display of Mathcad output).

slower than the Fortran or Matlab we used in the past, the total time is much less with Mathcad.

"It's far faster to develop code in Mathcad. The particular advantage is the short time it takes to try out an idea. You can use formulae from textbooks and other publications without conversion to Fortran or C++. Mathcad also passes data readily to other programs such as Labview and Corel Draw; and we routinely use Axum to graph Mathcad data for internal and customer presentations."

To read more about the work of Ericsson Microwave Systems, visit its web site at **http://www.ericsson.se/microwave/** .